

Complete the square.

$$\left(\frac{b}{2a}\right)^2$$

$$ax^2 + bx + c = 0$$

$$x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = -\frac{c}{a} + \frac{b^2}{4a^2}$$

$$\sqrt{\left(x + \frac{b}{2a}\right)^2} = \sqrt{\frac{b^2 - 4ac}{4a^2}}$$

$$x + \frac{b}{2a} = \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

6-5 The Quadratic Formula and the Discriminant

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

ex 1

$$3x^2 + x - 1 = 0$$

$$a = 3$$

$$b = 1$$

$$c = -1$$

$$x = \frac{-1 \pm \sqrt{1 - 4(3)(-1)}}{2(3)}$$

$$x = \frac{-1 \pm \sqrt{13}}{6}$$

ex 2

$$5x^2 + 8 = -12x$$

$$5x^2 + 12x + 8 = 0$$

$$a = 5$$

$$b = 12$$

$$c = 8$$

$$x = \frac{-12 \pm \sqrt{144 - 4(5)(8)}}{2(5)}$$

$$\frac{-12 \pm \sqrt{-16}}{10}$$

$$\frac{-12 \pm 4i}{10}$$

$$x = \frac{-6 \pm 2i}{5}$$

The Discriminant

$$D = b^2 - 4ac$$

Determines the nature of the roots.

Three Cases

I. $D > 0$

2 \mathbb{R}

II. $D = 0$

1 \mathbb{R} double root

III. $D < 0$

2 imaginary roots

Ex

$$x^2 - 8x + 5 = 0$$

$$b^2 - 4ac$$

$$64 - 4(1)(5)$$

$$44$$

2 \mathbb{R} roots

irrational

Also able to determine if the roots are rational or irrational.

Rational

a, b, & c must be rational and D must be a perfect square (Real)

ex

$$x^2 + 10x + 25 = 0$$

$$D = 100 - 4(1)(25)$$

$$D = 0$$

one Real double root
rational

ex

$$x^2 - 4x + 13 = 0$$

$$D = 16 - 4(13)$$

2 Imaginary roots

Determine as much as you can

about the roots:

$$1. y^2 - 3y - 1 = 0$$

2 Real irr.

$$2. 3a^2 - 10a = -11$$

2 Imag.

$$3. 5x^2 + 2\sqrt{10}x + 2 = 0$$

1 double Real irr.

$$4. 3b^2 = 14b + 24$$

2 Real rat.

8/4

Find the value for k such that there are 2 imaginary roots

ex

$$5x^2 - 2x + k = 0$$

$$4 - 4(5)k < 0$$

$$4 < 20k$$

$$\frac{1}{5} < k$$

$$k > \frac{1}{5}$$

Find the value for k such that there is a double root

ex

$$3x^2 + 2x + k = 0$$

$$4 - 4(3)k = 0$$

$$4 = 12k$$

$$\frac{1}{3} = k$$

p318

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